This is an overview of ECE314 Lab 3.

In this lab, we will first talk about Bernoulli process, a particular kind of random process. Then we will talk about an interesting linkage between the poisson distribution and binomial distribution.

First, random process. In previous labs we have studied random variable. A random variable is a single value. A random process is simply a sequence of random variables, where the sequence is usually indexed by time. So when you simulate a random process, the result will be a sequence of numbers, not just one number.

In this lab, we will study a particular kind of random process, the Bernoulli process. In later labs, you will see other kinds of random processes.

A Bernoulli process, is a random process consisting of a sequence of independent Bernoulli random variables with the same distribution. You can think of it like this: you have a coin, which could be biased. You toss the coin repetitively. The random sequence you get is a Bernoulli process.

Now, suppose you simulate a Bernoulli process, and you get a sample path, which is a sequence of zeros and ones. We now discuss four different ways to describe such a sample path.

The first way is the most natural way. We use a sequence X to denote it, where Xt represents the bit, or the result at time t.

Suppose this is the generated sequence.

The second way to describe is to use the L sequence. The ith number in the L sequence is defined as the number of trials needed after the (i-1)th count to get the ith count. Here we say we have a count if we see a one.

S

C

The four different descriptions are equivalent, that is, knowing one gives you the other three. I will leave it to you to see why that is true. For example, if you know the sequence L, how do you get the sequences X, S and C? You can think about it.

Bernoulli process is covered in Section 2.6 of the ECE313 notes.

In the lab, you will write a program to simulate a Bernoulli process and generate these four equivalent sequences.

Finally, I will talk about an interesting relationship between the poisson distribution and the binomial distribution.

Let us review these two distributions first. Let X denote a Poission distributed random variable with parameter lambda. Let Y denote a binomial distributed random variable with parameter n and p.

The pmfs of these two distributions are provided here. The possible values of X are 0, 1, 2, 3, 4, etc, that is, all the non-negative integers. The possible values of Y are 0, 1, 2, … until n.

You probably have seen the binomial distribution several times. To help you get more familiar with the Poissoin distribution, let us write out the first several terms. To do that, you simply plug in the value of k equals to 0, 1, 2, 3, 4 etc into the pmf formula. And you get these terms.

If you are interested, you can verify that these terms add up to 1.

Now, back to the relationship between these two distributions. It turns out that under certain conditions, these two distributions are very close to each other. By being close, I mean that if plot their pmfs and CDFs, they will look very similar.

Now, what are the conditions? First, n is large, p is small, and lambda is close to the product of n and p.

We won’t spend time explaining why this is true. If you are interested, you can take a look at Section 2.7 of the ECE313 notes. There is a proof of this result. In the lab, you will write a program to observe this phenomenon.

Now, the reason that we care about this is that it is often useful when we need to make an approximation in practice and in research. In particular, if you take a look at the pmf of the biniomial distribution, it has two parameters, n and p, and its expression has a combinatorial factor and a power of p times a power of 1-p. This expression is often cumbersome to work with. However, in the pmf expression of the Poisson distribution, you only has one parameter lambda and its form is often easier to work with mathematically.

Let us take an example. ……..